

Distribution amplitudes of Σ and Λ and their electromagnetic form factors

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Abstract

Based on QCD conformal partial wave expansion to leading order conformal spin accuracy, we present the light-cone distribution amplitudes (DAs) of Σ and Λ baryons up to twist 6. It is concluded that fourteen independent DAs are needed to describe the valence three-quark states of the baryons at small transverse separations. The nonperturbative parameters relevant to the DAs are determined within the framework of QCD sum rule method. With the obtained DAs, a simple investigation on the electromagnetic form factors of these baryons are given. The magnetic moments of the baryons are estimated by fitting the magnetic form factor with the dipole formula.

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1 Introduction

Theory of hard exclusive processes in QCD has been studied extensively for several decades. The investigation for these transitions provides unmatched opportunities to understand the hadron structure, and the theoretical method for the calculation of these processes was developed early in 1970's [1, 2]. In the model of the hard exclusive process, the concept of distribution amplitudes (DAs), which are the fundamental nonperturbative functions describing the hadronic structure, was introduced. The DAs, physically speaking, describe the decomposition of the hadron momentum in parton configurations, which is important to make the QCD description of hard exclusive reactions quantitative.

DAs of mesons have been investigated extensively in the past, some of which have been done to high twist accuracy [3, 4, 5, 6, 7]. However, the corresponding studies on baryons received less attention due to their relatively complex structure, and the existing investigations were mainly focused on the nucleon (See [8] for a review). DAs of the nucleon and other octet baryons were firstly calculated within the QCD sum rule framework on the moments in Ref. [9]. A systematic study on the nucleon DAs were provided in Ref. [10], in which the DAs of the nucleon are investigated up to twist 6. In the paper [11], the axial vector higher twist DAs of the Λ baryon were given to leading order conformal spin accuracy. Recently Ref. [12] gave DAs of Λ_b in the heavy quark limit, and Ref. [13] offered a complete analysis of the one-loop renormalization of twist-4 operators of the nucleon. In the literature [14], the author presented a description on DAs of helicity $\lambda = 3/2$ baryons with a new approach. Actually, the investigation for DAs of Σ and Λ baryons was firstly done by Chernyak *et al.* in Ref. [9], in which they calculated the DAs up to leading twist order only. Nevertheless, more detailed description of the internal structures of these baryons needs information on higher twist DAs. The present work is devoted to give an investigation on Σ and Λ DAs up to twist 6, and then to study their electromagnetic form factors as an application.

Higher twist contributions to DAs come from several physical origins. The first con-

tribution is from “bad” components in the wave function and in particular of components with “wrong” spin projection. The second one comes from transverse motion of quarks in the leading twist components. Finally, higher Fock states with additional gluons or quark/antiquark pairs also contribute to the DAs. It has been known that for mesons, contributions due to “bad” components in the quark-antiquark DAs can be described in terms of higher Fock states by equations of motion [3, 5]. Since quark-antiquark-gluon matrix elements between the vacuum and the meson state are numerically small, contributions of “bad” components to mesonic DAs are small enough. However, things are different for baryons because equations of motion are not sufficient to eliminate the higher-twist three-quark system with additional gluons. At the same time, matrix elements of higher twist three-quark operators are large compared with the leading one. Thus the first contribution is assumed to dominate the DAs rather than the other two origins. For this reason we only consider contributions coming from “bad” components in the decomposition of the Lorentz structure in this paper.

The usual description of DAs is based on the conformal symmetry of the massless QCD Lagrangian for dynamics dominated on the light-cone. DAs with definite twist can be expanded by partial wave functions with the specific conformal spin j . The conformal spin of a quark is defined as $j = (l + s)/2$, where l is the canonical dimension of the quark and s is its spin. For a composite particle, contributions of the higher order conformal spin $j + n$ ($n = 0, 1, 2, \dots$) are given by the leading contribution multiplied by polynomials which are orthogonal over the leading weight function. In this paper, DAs of Σ and Λ baryons are investigated on the conformal partial wave expansion approach. At first glance, the mass terms of the s quark break the conformal symmetry of the QCD Lagrangian explicitly, and the $SU(3)$ breaking corrections seem difficult to be included. However, this is not a problem as argued by Ball *et al.* in the case of mesons [5]. The transverse wave functions in the conformal expansion are dependent on the scale relevant to the process. If the s quark mass is smaller than the QCD scale, the transverse-momentum dependence is not affected by the quark mass. Therefore the conformal expansion of the DAs can be carried out

safely. In this work, we utilize the method proposed in Ref. [10] and consider the $SU(3)$ flavor symmetry breaking corrections. In fact, the $SU(3)$ flavor symmetry breaking results in two different effects, which are isospin symmetry breaking and the corrections to the nonperturbative parameters. Unlike the case of nucleon, in which the isospin symmetry leads to symmetric relationships to reduce the number of the independent DAs to 8, in our case the description of Λ and Σ baryons needs fourteen independent DAs that are expanded in operators with increasing conformal spin. With equations of motion, the parameters of the conformal expansion are expressed in terms of local nonperturbative parameters, which need to be determined by nonperturbative QCD methods. In the calculation, we expand the DAs to leading order conformal spin accuracy, and use QCD sum rules to determine the nonperturbative parameters.

DAs provide large opportunities to investigate processes connected with the baryons in the framework of light-cone sum rule (LCSR) since they are fundamental input parameters in this framework [11, 15, 16, 17, 18, 19]. LCSR is a developed nonperturbative QCD method that includes the traditional à la SVZ sum rule [20] technique and the theory of hard exclusive processes. The main idea of LCSR is to expand the products of currents near the light-cone, and the nonperturbative effects are described by DAs rather than condensates in the traditional QCD sum rule [21, 22, 23]. As a simple application, the electromagnetic (EM) form factors of the baryons are examined in LCSR with the obtained DAs. EM form factors are fundamental objects for understanding the inner structure of the hadron. As there are no experimental data available on Σ and Λ EM form factors, it is instructive and necessary to give an investigation theoretically. In the experimental point of view, the EM form factors can be described by the electric and magnetic Sachs form factors, and the magnetic Sachs form factor at zero momentum transfer defines the magnetic moment of the baryon. It is assumed that the dependence of the magnetic form factor on the momentum transfer can be expressed by the dipole formula, therefore after fitting the magnetic form factor by the dipole formula, we give estimations on the magnetic moments of the Σ baryons.

The paper is organized as follows. Section 2 presents notations and definitions of the baryon DAs up to twist 6 by the matrix element of the three-quark operator between the hadron state and the vacuum. This section also gives properties of DAs from the symmetry relationships. Section 3 gives the leading-order conformal expansion of DAs based on the conformal invariance of the Lagrangian. The DAs are simplified to the nonperturbative parameters which can be calculated in the framework of the QCD sum rule. Section 4 is devoted to derive the QCD sum rules for the nonperturbative parameters related to the DAs and then present the numerical analysis. Section 5 is a simple application of the obtained DAs to investigate the EM form factors of the baryons. We give in this section the dependence of the EM form factors on the momentum transfer. After fitting the results by the dipole formula, the magnetic moments of the baryons are estimated numerically. Summary and conclusion are given in section 6.

2 Definitions of the light-cone distribution amplitudes

2.1 General classification

In the $SU(3)$ flavor symmetry limit, light-cone distribution amplitudes of octet baryons with quantum number $J^P = \frac{1}{2}^+$ can be expressed in terms of the matrix element of the gauge-invariant operators sandwiched between the vacuum and the baryon state:

$$\langle 0 | \epsilon^{ijk} q_{1\alpha}^i(a_1 z) q_{2\beta}^j(a_2 z) q_{3\gamma}^k(a_3 z) | X(P) \rangle, \quad (1)$$

where letters α, β, γ refer to Lorentz indices and i, j, k refer to color indices, q_i denote the quark fields, z is a light-like vector which satisfies $z^2 = 0$ and a_i are real numbers denoting coordinates of valence quarks.

In view of the Lorentz covariance, spin and parity of the baryons, the general decomposition of the matrix element (1) is written as [10, 11]:

$$\begin{aligned} & 4 \langle 0 | \epsilon^{ijk} q_{1\alpha}^i(a_1 z) q_{2\beta}^j(a_2 z) q_{3\gamma}^k(a_3 z) | X(P) \rangle \\ &= \mathcal{S}_1 M C_{\alpha\beta} (\gamma_5 X)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta} (\not{z} \gamma_5 X)_\gamma + \mathcal{P}_1 M (\gamma_5 C)_{\alpha\beta} X_\gamma + \mathcal{P}_2 M^2 (\gamma_5 C)_{\alpha\beta} (\not{z} X)_\gamma \end{aligned}$$

$$\begin{aligned}
& +\mathcal{V}_1 (\mathcal{P}C)_{\alpha\beta} (\gamma_5 X)_\gamma + \mathcal{V}_2 M (\mathcal{P}C)_{\alpha\beta} (\not{z}\gamma_5 X)_\gamma + \mathcal{V}_3 M (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 X)_\gamma \\
& +\mathcal{V}_4 M^2 (\not{z}C)_{\alpha\beta} (\gamma_5 X)_\gamma + \mathcal{V}_5 M^2 (\gamma_\mu C)_{\alpha\beta} (i\sigma^{\mu\nu} z_\nu \gamma_5 X)_\gamma + \mathcal{V}_6 M^3 (\not{z}C)_{\alpha\beta} (\not{z}\gamma_5 X)_\gamma \\
& +\mathcal{A}_1 (\mathcal{P}\gamma_5 C)_{\alpha\beta} X_\gamma + \mathcal{A}_2 M (\mathcal{P}\gamma_5 C)_{\alpha\beta} (\not{z}X)_\gamma + \mathcal{A}_3 M (\gamma_\mu \gamma_5 C)_{\alpha\beta} (\gamma^\mu X)_\gamma \\
& +\mathcal{A}_4 M^2 (\not{z}\gamma_5 C)_{\alpha\beta} X_\gamma + \mathcal{A}_5 M^2 (\gamma_\mu \gamma_5 C)_{\alpha\beta} (i\sigma^{\mu\nu} z_\nu X)_\gamma + \mathcal{A}_6 M^3 (\not{z}\gamma_5 C)_{\alpha\beta} (\not{z}X)_\gamma \\
& +\mathcal{T}_1 (P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 X)_\gamma + \mathcal{T}_2 M (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma_5 X)_\gamma \\
& +\mathcal{T}_3 M (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 X)_\gamma + \mathcal{T}_4 M (P^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 X)_\gamma \\
& +\mathcal{T}_5 M^2 (z^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 X)_\gamma + \mathcal{T}_6 M^2 (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\not{z}\gamma_5 X)_\gamma \\
& +\mathcal{T}_7 M^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \not{z}\gamma_5 X)_\gamma + \mathcal{T}_8 M^3 (z^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 X)_\gamma ,
\end{aligned} \tag{2}$$

where X_γ is the spinor of the baryon, C is the charge conjugation matrix and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. In the present investigations X_γ denotes baryons with the quantum number $I(J^P) = 1(\frac{1}{2}^+)$ for Σ^\pm and $I(J^P) = 0(\frac{1}{2}^+)$ for Λ (I is the isospin, J is the total angular momentum and P is the parity). All the functions \mathcal{S}_i , \mathcal{P}_i , \mathcal{A}_i , \mathcal{V}_i and \mathcal{T}_i depend on the scalar product $P \cdot z$.

The ‘‘calligraphic’’ invariant functions in Eq. (2) do not have a definite twist however, thus the twist classification needs to be carried out in the infinite momentum frame. Here we introduce the second auxiliary light-like vector:

$$p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{M^2}{p \cdot z}, \quad p^2 = 0, \tag{3}$$

so that $P \rightarrow p$ if the baryon mass can be neglected. In the infinite momentum frame, the baryon is assumed to move in the positive \mathbf{e}_z direction, hence p^+ and z^- are the only non-vanishing components of p and z . In this frame, terms on twist can be classified in powers of p^+ , and the baryon spinor X_γ is decomposed into ‘‘large’’ and ‘‘small’’ components X_γ^+ and X_γ^- :

$$X_\gamma(P, \lambda) = \frac{1}{2p \cdot z} (\not{p} \not{z} + \not{z} \not{p}) = X_\gamma^+(P, \lambda) + X_\gamma^-(P, \lambda), \tag{4}$$

where the projection operators

$$\Lambda^+ = \frac{\not{p} \not{z}}{2p \cdot z}, \quad \Lambda^- = \frac{\not{z} \not{p}}{2p \cdot z} \quad (5)$$

project the spinor onto the “plus” and “minus” components. From the Dirac equation $\not{p}X(P) = MX(P)$, we get the following useful relations:

$$\not{p}X(P) = MX^+(P), \quad \not{z}X(P) = \frac{2p \cdot z}{M}X^-(P). \quad (6)$$

Using the explicit expressions for $X(P)$, it is easy to see that $\Lambda^+X = X^+ \sim \sqrt{p^+}$ while $\Lambda^-X = X^- \sim 1/\sqrt{p^+}$. By the twist counts in terms of $1/p^+$ the definition of light-cone DAs with a definite twist is given as

$$\begin{aligned} & 4\langle 0 | \epsilon^{ijk} q_{1\alpha}^i(a_1 z) q_{2\beta}^j(a_2 z) q_{3\gamma}^k(a_3 z) | X(P) \rangle \\ &= S_1 MC_{\alpha\beta} (\gamma_5 X^+)_{\gamma} + S_2 MC_{\alpha\beta} (\gamma_5 X^-)_{\gamma} + P_1 M (\gamma_5 C)_{\alpha\beta} X_{\gamma}^+ + P_2 M (\gamma_5 C)_{\alpha\beta} X_{\gamma}^- \\ &+ V_1 (\not{p}C)_{\alpha\beta} (\gamma_5 X^+)_{\gamma} + V_2 (\not{p}C)_{\alpha\beta} (\gamma_5 X^-)_{\gamma} + \frac{V_3}{2} M (\gamma_{\perp} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^+)_{\gamma} \\ &+ \frac{V_4}{2} M (\gamma_{\perp} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^-)_{\gamma} + V_5 \frac{M^2}{2pz} (\not{z}C)_{\alpha\beta} (\gamma_5 X^+)_{\gamma} + \frac{M^2}{2pz} V_6 (\not{z}C)_{\alpha\beta} (\gamma_5 X^-)_{\gamma} \\ &+ A_1 (\not{p} \gamma_5 C)_{\alpha\beta} X_{\gamma}^+ + A_2 (\not{p} \gamma_5 C)_{\alpha\beta} X_{\gamma}^- + \frac{A_3}{2} M (\gamma_{\perp} \gamma_5 C)_{\alpha\beta} (\gamma^{\perp} X^+)_{\gamma} \\ &+ \frac{A_4}{2} M (\gamma_{\perp} \gamma_5 C)_{\alpha\beta} (\gamma^{\perp} X^-)_{\gamma} + A_5 \frac{M^2}{2pz} (\not{z} \gamma_5 C)_{\alpha\beta} X_{\gamma}^+ + \frac{M^2}{2pz} A_6 (\not{z} \gamma_5 C)_{\alpha\beta} X_{\gamma}^- \\ &+ T_1 (i\sigma_{\perp p} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^+)_{\gamma} + T_2 (i\sigma_{\perp p} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^-)_{\gamma} + T_3 \frac{M}{pz} (i\sigma_{pz} C)_{\alpha\beta} (\gamma_5 X^+)_{\gamma} \\ &+ T_4 \frac{M}{pz} (i\sigma_{zp} C)_{\alpha\beta} (\gamma_5 X^-)_{\gamma} + T_5 \frac{M^2}{2pz} (i\sigma_{\perp z} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^+)_{\gamma} \\ &+ \frac{M^2}{2pz} T_6 (i\sigma_{\perp z} C)_{\alpha\beta} (\gamma^{\perp} \gamma_5 X^-)_{\gamma} + M \frac{T_7}{2} (\sigma_{\perp \perp'} C)_{\alpha\beta} (\sigma^{\perp \perp'} \gamma_5 X^+)_{\gamma} \\ &+ M \frac{T_8}{2} (\sigma_{\perp \perp'} C)_{\alpha\beta} (\sigma^{\perp \perp'} \gamma_5 X^-)_{\gamma}, \end{aligned} \quad (7)$$

where an obvious notation $\sigma_{pz} = \sigma^{\mu\nu} p_{\mu} z_{\nu}$, etc., is used as a shorthand and \perp stands for the projection transverse to z, p , e.g. $\gamma_{\perp} \gamma^{\perp} = \gamma^{\mu} g_{\mu\nu}^{\perp} \gamma^{\nu}$ with $g_{\mu\nu}^{\perp} = g_{\mu\nu} - (p_{\mu} z_{\nu} + z_{\mu} p_{\nu})/pz$. The DAs $F = S_i, P_i, V_i, A_i, T_i$ with a definite twist are classified in Table 1. Each distribution

amplitude F_i can be represented as

$$F(a_i p \cdot z) = \int \mathcal{D}x e^{-ipz \sum_i x_i a_i} F(x_i), \quad (8)$$

where the dimensionless variables x_i , which satisfy the relation $0 < x_i < 1$, $\sum_i x_i = 1$, correspond to the longitudinal momentum fractions carried by the quarks inside the baryon. The integration measure is defined as

$$\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1). \quad (9)$$

Comparing Eq. (2) and (7), the invariant functions $\mathcal{S}_i, \mathcal{P}_i, \mathcal{V}_i, \mathcal{A}_i, \mathcal{T}_i$ can be expressed in terms of the DAs S_i, P_i, V_i, A_i, T_i with a definite twist.

For scalar and pseudo-scalar distributions the following relations hold:

$$\begin{aligned} \mathcal{S}_1 &= S_1, & 2p \cdot z \mathcal{S}_2 &= S_1 - S_2, \\ \mathcal{P}_1 &= P_1, & 2p \cdot z \mathcal{P}_2 &= P_2 - P_1, \end{aligned} \quad (10)$$

for vector distributions:

$$\begin{aligned} \mathcal{V}_1 &= V_1, & 2p \cdot z \mathcal{V}_2 &= V_1 - V_2 - V_3, \\ 2\mathcal{V}_3 &= V_3, & 4p \cdot z \mathcal{V}_4 &= -2V_1 + V_3 + V_4 + 2V_5, \\ 4p \cdot z \mathcal{V}_5 &= V_4 - V_3, & (2p \cdot z)^2 \mathcal{V}_6 &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, \end{aligned} \quad (11)$$

for axial vector distributions:

$$\begin{aligned} \mathcal{A}_1 &= A_1, & 2p \cdot z \mathcal{A}_2 &= -A_1 + A_2 - A_3, \\ 2\mathcal{A}_3 &= A_3, & 4p \cdot z \mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5, \\ 4p \cdot z \mathcal{A}_5 &= A_3 - A_4, & (2p \cdot z)^2 \mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6, \end{aligned} \quad (12)$$

and, finally, for tensor distributions:

$$\begin{aligned} \mathcal{T}_1 &= T_1, & 2p \cdot z \mathcal{T}_2 &= T_1 + T_2 - 2T_3, \\ 2\mathcal{T}_3 &= T_7, & 2p \cdot z \mathcal{T}_4 &= T_1 - T_2 - 2T_7, \\ 2p \cdot z \mathcal{T}_5 &= -T_1 + T_5 + 2T_8, & (2p \cdot z)^2 \mathcal{T}_6 &= 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \\ 4p \cdot z \mathcal{T}_7 &= T_7 - T_8, & (2p \cdot z)^2 \mathcal{T}_8 &= -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8. \end{aligned} \quad (13)$$

For $\Sigma^{+(-)}$, the identity of the two $u(d)$ quarks in the baryon gives symmetry properties of the DAs. The Lorentz decomposition on the γ -matrix structure implies that the vector and tensor DAs are symmetric, whereas the scalar, pseudo-scalar and axial-vector DAs are antisymmetric under the interchange of the two $u(d)$ quarks:

$$\begin{aligned}
V_i(1, 2, 3) &= V_i(2, 1, 3), & T_i(1, 2, 3) &= T_i(2, 1, 3), \\
S_i(1, 2, 3) &= -S_i(2, 1, 3), & P_i(1, 2, 3) &= -P(2, 1, 3), \\
A_i(1, 2, 3) &= -A(2, 1, 3).
\end{aligned} \tag{14}$$

The ‘‘calligraphic’’ structures in Eq. (2) have the similar relationships.

For Λ , the isospin symmetry leads to similar relationships that the vector and tensor DAs are antisymmetric, while the scalar, pseudoscalar and axial-vector DAs are symmetric:

$$\begin{aligned}
V_i(1, 2, 3) &= -V_i(2, 1, 3), & T_i(1, 2, 3) &= -T_i(2, 1, 3), \\
S_i(1, 2, 3) &= S_i(2, 1, 3), & P_i(1, 2, 3) &= P(2, 1, 3), \\
A_i(1, 2, 3) &= A(2, 1, 3).
\end{aligned} \tag{15}$$

2.2 Representation in terms of chiral fields

This subsection gives the DAs’ representation in terms of chiral fields. The discussion is mainly about Σ^+ baryon, and the counterparts of the others are similar. In terms of quark fields with definite chirality:

$$q^{\uparrow(\downarrow)} = \frac{1}{2}(1 \pm \gamma_5)q, \tag{16}$$

the DAs can be interpreted transparently. Projection on the state where the spins of the two u quarks are antiparallel, that is $u^\uparrow u^\downarrow$, singles out vector and axial vector amplitudes, while the two u quarks are parallel, $u^\uparrow u^\uparrow$ and $u^\downarrow u^\downarrow$, singles out scalar, pseudo-scalar and tensor structures. Similar as expressions in Ref. [10], the DAs can be defined in terms of chiral fields. The leading twist-3 distribution amplitude can be defined as:

$$\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z) \right) \not{z} s_k^\uparrow(a_3 z) | P \rangle = -\frac{1}{2} p z \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-i p z \Sigma x_i a_i} \Phi_3(x_i),$$

(17)

and the distributions for twist-4 are:

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{z} u_j^\downarrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= -\frac{1}{2} p z \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Phi_4(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{z} \gamma_\perp \not{p} u_j^\downarrow(a_2 z) \right) \gamma^\perp \not{s}_k^\downarrow(a_3 z) | P \rangle &= -p z M \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Psi_4(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{p} \not{z} u_j^\uparrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= \frac{1}{2} p z M \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Xi_4(x_i),
\end{aligned} \tag{18}$$

and the distributions for twist-5 are similarly written as:

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{p} u_j^\downarrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= -\frac{1}{4} M^2 \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Phi_5(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{p} \gamma_\perp \not{z} u_j^\downarrow(a_2 z) \right) \gamma^\perp \not{s}_k^\downarrow(a_3 z) | P \rangle &= -p z M \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Psi_5(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{z} \not{p} u_j^\uparrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= \frac{1}{2} p z M \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Xi_5(x_i),
\end{aligned} \tag{19}$$

and finally the twist-6 one can be expressed as:

$$\langle 0 | \epsilon^{ijk} \left(u_i^\uparrow(a_1 z) C \not{p} u_j^\downarrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle = -\frac{1}{4} M^2 \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} \Phi_6(x_i). \tag{20}$$

Due to $SU(3)$ flavor symmetry breaking effects, there are not similar relationships as that in Ref. [10] from the isospin symmetry to reduce the number of the independent DAs.

So the following additional chiral fields representations are needed to get all the DAs:

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} \left(u_i^\downarrow(a_1 z) C \not{p} \not{z} u_j^\downarrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= \frac{1}{2} p z M \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} (S_1 - P_1 + T_3 + T_7), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\downarrow(a_1 z) C \not{z} \not{p} u_j^\downarrow(a_2 z) \right) \not{s}_k^\uparrow(a_3 z) | P \rangle &= \frac{1}{2} p z M \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \sum x_i a_i} (S_2 - P_2 - T_4 + T_8),
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} \left(u_i^\dagger(a_1 z) C i \sigma_{\perp z} u_j^\dagger(a_2 z) \right) \gamma^\perp \not{s}_k^\dagger(a_3 z) | P \rangle &= -2pz \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \Sigma x_i a_i} T_1(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\dagger(a_1 z) C i \sigma_{\perp z} u_j^\dagger(a_2 z) \right) \gamma^\perp \not{p} s_k^\dagger(a_3 z) | P \rangle &= -2pz \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \Sigma x_i a_i} T_2(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\dagger(a_1 z) C i \sigma_{\perp p} u_j^\dagger(a_2 z) \right) \gamma^\perp \not{s}_k^\dagger(a_3 z) | P \rangle &= -M^2 \not{z} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \Sigma x_i a_i} T_5(x_i), \\
\langle 0 | \epsilon^{ijk} \left(u_i^\dagger(a_1 z) C i \sigma_{\perp p} u_j^\dagger(a_2 z) \right) \gamma^\perp \not{p} s_k^\dagger(a_3 z) | P \rangle &= -M^2 \not{p} \Sigma^{+\uparrow} \int \mathcal{D}x e^{-ipz \Sigma x_i a_i} T_6(x_i).
\end{aligned} \tag{22}$$

The twist classification of these additional DAs are shown in Table 1. The following denotations $(S_1 - P_1 + T_3 + T_7)(x_i) \equiv \Xi'_4(x_i)$, and $(S_2 - P_2 - T_4 + T_8)(x_i) \equiv \Xi'_5(x_i)$ are adopted for convenience.

The similar relationships hold for the Λ baryon under the exchange u, u, s to u, d, s , and for the Σ^- baryon under the exchange u, u, s to d, d, s .

3 Conformal expansion

The spirit of the conformal expansion of distribution amplitudes is similar to the partial wave expansion of a wave function in quantum mechanics. The idea is to use the conformal symmetry of the massless QCD Lagrangian to study the DAs, which allows to separate longitudinal degrees of freedom from transverse ones [3, 7, 10]. The transverse coordinates are replaced by the renormalization scale, which is determined by the renormalization group. The dependence on longitudinal momentum fractions, which is living on the light cone, is taken into account by a set of orthogonal polynomials that form an irreducible representation of the collinear subgroup $SL(2, R)$ of the conformal group. As to leading logarithmic accuracy, the renormalization group equations are driven by tree-level counter terms, they have the conformal symmetry. This leads to the fact that components of the DAs with different conformal spin do not mix under renormalization to this accuracy.

The $SL(2, R)$ group is governed by four generators \mathbf{P}_+ , \mathbf{M}_{-+} , \mathbf{D} and \mathbf{K}_- , where the definitions are used for a vector A : $A_+ = A_\mu z^\mu$ and $A_- = A_\mu p^\mu / p \cdot z$. The four generators

\mathbf{P}_μ , \mathbf{K}_μ , \mathbf{D} and $\mathbf{M}_{\mu\nu}$ are the translation, special conformal transformation, dilation and Lorentz generators, respectively. The generators of the collinear subgroup $SL(2, R)$ can be described by the following four operators:

$$\mathbf{L}_+ = -i\mathbf{P}_+, \quad \mathbf{L}_- = \frac{i}{2}\mathbf{K}_-, \quad \mathbf{L}_0 = -\frac{i}{2}(\mathbf{D} - \mathbf{M}_{-+}), \quad \mathbf{E} = i(\mathbf{D} + \mathbf{M}_{-+}). \quad (23)$$

For a field living on the light cone $\Phi(z)$, the acting of the above generators on it yields the following relations:

$$\begin{aligned} [\mathbf{L}^2, \Phi(z)] &= j(j-1)\Phi(z), & [\mathbf{E}, \Phi(z)] &= (l-s)\Phi(z), \\ [\mathbf{E}, \mathbf{L}^2] &= 0, & [\mathbf{E}, \mathbf{L}_0] &= 0, \end{aligned} \quad (24)$$

and

$$\mathbf{L}^2 = \mathbf{L}_0^2 - \mathbf{L}_0 + \mathbf{L}_+\mathbf{L}_-, \quad (25)$$

where $j = (l + s)/2$ is called the conformal spin and $t = l - s$ is the twist. In above notations, l is the canonical dimension of the quark field and s is the quark spin projection on the light-cone. The role of the generator \mathbf{E} is analogous to the Hamiltonian in quantum mechanics, and the twist corresponds to the eigenvalue of the Hamiltonian. For a given twist distribution amplitude, it can be expanded by the conformal partial wave functions that are the eigenstates of \mathbf{L}^2 and L_0 .

For multi-quark states, we need to deal with the problem of summation of conformal spins, and here the group is non-compact. The distribution amplitude with the lowest conformal spin $j_{min} = j_1 + j_2 + j_3$ of a three-quark state is [3, 4]

$$\Phi_{as}(x_1, x_2, x_3) = \frac{\Gamma[2j_1 + 2j_2 + 2j_3]}{\Gamma[2j_1]\Gamma[2j_2]\Gamma[2j_3]} x_1^{2j_1-1} x_2^{2j_2-1} x_3^{2j_3-1}. \quad (26)$$

Contributions with higher conformal spin $j = j_{min} + n$ ($n = 1, 2, \dots$) are given by Φ_{as} multiplied by polynomials that are orthogonal over the weight function (26). In this paper, we just consider DAs to leading order conformal spin accuracy. For DAs in Table 1, we give their conformal expansion:

$$\Phi_3(x_i) = 120x_1x_2x_3\phi_3^0(\mu), \quad T_1(x_i) = 120x_1x_2x_3\phi_3^{\prime 0}(\mu), \quad (27)$$

for twist 3 and

$$\begin{aligned}
\Phi_4(x_i) &= 24x_1x_2\phi_4^0(\mu), & \Psi_4(x_i) &= 24x_1x_3\psi_4^0(\mu), \\
\Xi_4(x_i) &= 24x_2x_3\xi_4^0(\mu), & \Xi'_4(x_i) &= 24x_2x_3\xi_4'^0(\mu), \\
T_2(x_i) &= 24x_1x_2\phi_4'(\mu), & &
\end{aligned} \tag{28}$$

for twist 4 and

$$\begin{aligned}
\Phi_5(x_i) &= 6x_3\phi_5^0(\mu), & \Psi_5(x_i) &= 6x_2\psi_5^0(\mu), \\
\Xi_5(x_i) &= 6x_1\xi_5^0(\mu), & \Xi'_5(x_i) &= 6x_1\xi_5'^0(\mu), \\
T_5(x_i) &= 6x_3\phi_5'(\mu), & &
\end{aligned} \tag{29}$$

for twist 5 and

$$\Phi_6(x_i) = 2\phi_6^0(\mu), \quad T_6(x_i) = 2\phi_6'(\mu). \tag{30}$$

for twist 6. There are altogether 14 parameters which can be determined by the equations of motion.

3.1 DAs of the Σ baryon

The normalization of the DAs of Σ^+ are determined by matrix element of the local three-quark operator. The Lorentz decomposition of the matrix element can be expressed explicitly as follows:

$$\begin{aligned}
4\langle 0|\epsilon^{ijk}u_\alpha^i(0)u_\beta^j(0)s_\gamma^k(0)|\Sigma^+(P)\rangle &= \mathcal{V}_1^0(\mathcal{P}C)_{\alpha\beta}(\gamma_5\Sigma^+)_\gamma + \mathcal{V}_3^0(\gamma_\mu C)_{\alpha\beta}(\gamma_\mu\gamma_5\Sigma^+)_\gamma \\
&+ \mathcal{T}_1^0(P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Sigma^+)_\gamma + \mathcal{T}_3^0M(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\gamma_5\Sigma^+)_\gamma. \tag{31}
\end{aligned}$$

Similar to definitions in Ref. [10], the above four parameters can be expressed by the following matrix elements:

$$\langle 0|\epsilon^{ijk}[u^i(0)C\not{z}u^j(0)]\gamma_5\not{z}s^k(0)|P\rangle = f_{\Sigma^+}(p\cdot z)\not{z}\Sigma^+(P),$$

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_\mu u^j(0)] \gamma_5 \gamma^\mu s^k(0) | P \rangle &= \lambda_1 M \Sigma^+(P), \\
\langle 0 | \epsilon^{ijk} [u^i(0) C \sigma_{\mu\nu} u^j(0)] \gamma_5 \sigma^{\mu\nu} s^k(0) | P \rangle &= \lambda_2 M \Sigma^+(P), \\
\langle 0 | \epsilon^{ijk} [u^i(0) C i q^\nu \sigma_{\mu\nu} u^j(0)] \gamma_5 \gamma_\mu s^k(0) | P \rangle &= \lambda_3 M \not{q} \Sigma^+(P).
\end{aligned} \tag{32}$$

As mentioned above, there are no relations derived from isospin symmetry, so four but not three matrix elements are needed to determine the four parameters $\mathcal{V}_1^0, \mathcal{V}_3^0, \mathcal{T}_1^0$, and \mathcal{T}_3^0 . After a simple calculation, we arrive at the following expressions of $\mathcal{V}_1^0, \mathcal{V}_3^0, \mathcal{T}_1^0$, and \mathcal{T}_3^0 with the four parameters defined in Eqs. (32):

$$\begin{aligned}
\mathcal{V}_1^0 &= f_{\Sigma^+}, & \mathcal{V}_3^0 &= \frac{1}{4}(f_{\Sigma^+} - \lambda_1), \\
\mathcal{T}_1^0 &= \frac{1}{6}(4\lambda_3 - \lambda_2), & \mathcal{T}_3^0 &= \frac{1}{12}(2\lambda_3 - \lambda_2).
\end{aligned} \tag{33}$$

At the same time, coefficients of operators in Eqs. (27)-(30) can be expressed to leading order conformal spin accuracy as

$$\begin{aligned}
\phi_3^0 &= \phi_6^0 = f_{\Sigma^+}, & \psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_{\Sigma^+} - \lambda_1), \\
\phi_4^0 &= \phi_5^0 = \frac{1}{2}(f_{\Sigma^+} + \lambda_1), & \phi_3'^0 &= \phi_6'^0 = -\xi_5^0 = \frac{1}{6}(4\lambda_3 - \lambda_2), \\
\phi_4'^0 &= \xi_4^0 = \frac{1}{6}(8\lambda_3 - 3\lambda_2), & \phi_5'^0 &= -\xi_5'^0 = \frac{1}{6}\lambda_2, \\
\xi_4'^0 &= \frac{1}{6}(12\lambda_3 - 5\lambda_2).
\end{aligned} \tag{34}$$

3.2 DAs of the Λ baryon

The Lorentz decomposition of the local matrix element of Λ can be expressed explicitly as follows:

$$\begin{aligned}
4\langle 0 | \epsilon^{ijk} u_\alpha^i(0) d_\beta^j(0) s_\gamma^k(0) | \Lambda(P) \rangle &= \mathcal{S}_1^0 C_{\alpha\beta}(\gamma_5 \Lambda)_\gamma + \mathcal{P}_1^0 (\gamma_5 C)_{\alpha\beta} \Lambda_\gamma \\
&+ \mathcal{A}_1^0 (\not{P} \gamma_5 C)_{\alpha\beta} \Lambda_\gamma + \mathcal{A}_3^0 M (\gamma_\mu \gamma_5 C)_{\alpha\beta} (\gamma^\mu \Lambda)_\gamma.
\end{aligned} \tag{35}$$

In order to get the above parameters, the following matrix elements are used:

$$\langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_5 \not{z} d^j(0)] \not{z} s^k(0) | P \rangle = f_\Lambda (p \cdot z) \not{z} \Lambda(P),$$

$$\begin{aligned}
\langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_5 \gamma_\mu d^j(0)] \gamma^\mu s^k(0) | P \rangle &= \lambda_1 M \Lambda(P), \\
\langle 0 | \epsilon^{ijk} [u^i(0) C \gamma_5 d^j(0)] s^k(0) | P \rangle &= \lambda_2 M \Lambda(P), \\
\langle 0 | \epsilon^{ijk} [u^i(0) C d^j(0)] \gamma_5 s^k(0) | P \rangle &= \lambda_3 M^2 \Lambda(P).
\end{aligned} \tag{36}$$

A simple calculation leads to the following relationships:

$$\begin{aligned}
\mathcal{A}_1^0 &= f_\Lambda, & \mathcal{A}_3^0 &= -\frac{1}{4}(f_\Lambda - \lambda_1), \\
\mathcal{P}_1^0 &= \lambda_2, & \mathcal{S}_1^0 &= \lambda_3.
\end{aligned} \tag{37}$$

To leading order of the conformal spin expansion, coefficients of operators in Eqs. (27)-(30) for Λ can be expressed as

$$\begin{aligned}
\phi_3^0 &= \phi_6^0 = -f_\Lambda, & \phi_4^0 &= \phi_5^0 = -\frac{1}{2}(f_\Lambda + \lambda_1), \\
\psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_\Lambda - \lambda_1), & \xi_4^0 &= \xi_5^0 = \lambda_2 + \lambda_3, \\
\xi_4'^0 &= \xi_5'^0 = \lambda_3 - \lambda_2.
\end{aligned} \tag{38}$$

4 Determination of the parameters in QCD sum rules

4.1 QCD sum rules for Σ baryon

Determination of the nonperturbative parameters f_Σ , λ_1 , λ_2 and λ_3 can be done in the framework of QCD sum rule. The method is carried out from the following correlation functions for Σ^+ :

$$\Pi_i(q^2) = \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_i(x) \bar{j}_i(0) \} | 0 \rangle, \tag{39}$$

with the definitions of the currents:

$$j_1(x) = \epsilon^{ijk} [u^i(x) C \not{u}^j(x)] \gamma_5 \not{s}^k(x), \tag{40}$$

$$j_2(x) = \epsilon^{ijk} [u^i(x) C \gamma_\mu u^j(x)] \gamma_5 \gamma^\mu s^k(x), \tag{41}$$

$$j_3(x) = \epsilon^{ijk} [u^i(x) C \sigma_{\mu\nu} u^j(x)] \gamma_5 \sigma^{\mu\nu} s^k(x), \tag{42}$$

and the forth current:

$$j_4(x) = \epsilon^{ijk} [u^i(x) C i q^\nu \sigma_{\mu\nu} u^j(x)] \gamma_5 \gamma^\mu s^k(x). \quad (43)$$

Inserting the complete set of states with the same quantum numbers as those of Σ^+ , the hadronic representations of the correlation functions are given as

$$\begin{aligned} \Pi_1(q^2) &= 2f_{\Sigma^+}^2 (q \cdot z)^3 \not{z} \frac{1}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_1^h(s)}{s - q^2} ds, \\ \Pi_2(q^2) &= M^2 \lambda_1^2 \frac{\not{q} + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_2^h(s)}{s - q^2} ds, \\ \Pi_3(q^2) &= M^2 \lambda_2^2 \frac{\not{q} + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_3^h(s)}{s - q^2} ds, \\ \Pi_4(q^2) &= q^2 M^2 \lambda_3^2 \frac{\not{q} + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_4^h(s)}{s - q^2} ds. \end{aligned} \quad (44)$$

On the operator product expansion (OPE) side, condensates up to dimension 6 are taken into account. To give the sum rules, we utilize the dispersion relationship and assume the quark-hadron duality. After taking Borel transformation on both sides of the hadronic representation and QCD expansion, and equating the two sides, the final sum rules are given as follows:

$$4(2\pi)^4 f_{\Sigma^+}^2 e^{-\frac{M^2}{M_B^2}} = \frac{1}{5} \int_{m_s^2}^{s_0} s(1-x)^5 e^{-\frac{s}{M_B^2}} ds - \frac{b}{6} \int_{m_s^2}^{s_0} \frac{x(1-x)(1-2x)}{s} e^{-\frac{s}{M_B^2}} ds, \quad (45)$$

and

$$\begin{aligned} 4(2\pi)^4 \lambda_1^2 M^2 e^{-\frac{M^2}{M_B^2}} &= \int_{m_s^2}^{s_0} s^2 [(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] e^{-\frac{s}{M_B^2}} ds \\ &+ \frac{b}{6} \int_{m_s^2}^{s_0} (1-x)^2 e^{-\frac{s}{M_B^2}} ds + \frac{8}{3} a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4}\right) \\ &+ \frac{m_0^4 m_s^4}{16M_B^8} e^{-\frac{m^2}{M_B^2}} - 2a_s m_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds, \end{aligned} \quad (46)$$

and

$$\begin{aligned} 2(2\pi)^4 \lambda_2^2 M^2 e^{-M^2/M_B^2} &= - \int_{m_s^2}^{s_0} s^2 (-1 + 8x - 8x^3 + x^4 + 12x^2 \ln x) e^{-\frac{s}{M_B^2}} ds \\ &+ \frac{b}{3} \int_{m_s^2}^{s_0} (1-x)(4-7x) e^{-\frac{s}{M_B^2}} ds + 12m_s a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds, \end{aligned} \quad (47)$$

and

$$\begin{aligned}
(4\pi)^4 \lambda_3^2 M^2 e^{-M^2/M_B^2} &= \int_{m_s^2}^{s_0} s^2 \{[(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] \\
&\quad + \frac{1}{5}(1-x)^5\} e^{-\frac{s}{M_B^2}} ds + \frac{b}{12} \int_{m_s^2}^{s_0} (1-x)(11-5x-4x^2) e^{-\frac{s}{M_B^2}} ds \\
&\quad + 16a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4} + \frac{m_0^4 m_s^4}{12M_B^8}\right) e^{-\frac{m_s^2}{M_B^2}} \\
&\quad - 8m_s a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds. \tag{48}
\end{aligned}$$

where the notation $x = m_s^2/s$ is used, and the other parameters employed are the standard values: $a = -(2\pi)^2 \langle \bar{u}u \rangle = 0.55 \text{ GeV}^3$, $b = (2\pi)^2 \langle \alpha_s G^2/\pi \rangle = 0.47 \text{ GeV}^4$, $a_s = -(2\pi)^2 \langle \bar{s}s \rangle = 0.8a$, and $\langle \bar{u}g_c \sigma \cdot Gu \rangle = 0.8 \langle \bar{u}u \rangle$.

As the usual way of the sum rules, the auxiliary Borel parameter M_B^2 should have a proper range in which the results of the sum rules vary mildly with it. On the one hand the Borel parameter is expected to be large so that the higher order dimensional contributions are suppressed, and on the other hand the Borel parameter needs to be small enough to suppress the higher resonance contributions. Fig. 1 shows the dependence of the above parameters on the Borel parameter M_B^2 . The window of Borel parameter is choose as $1 \text{ GeV}^2 \leq M_B^2 \leq 2 \text{ GeV}^2$, in which our results are acceptable.

To determine the relative sign of f_{Σ^+} and λ_1 , we give the sum rule of $f_{\Sigma^+} \lambda_1^*$:

$$\begin{aligned}
4(2\pi)^4 f_{\Sigma^+} \lambda_1^* M e^{-\frac{M^2}{M_B^2}} &= -\frac{m_s}{3} \int_{m_s^2}^{s_0} s[(1-x)(3+13x-5x^2+x^3) + 12x \ln x] e^{-\frac{s}{M_B^2}} ds \\
&\quad - \frac{b}{6} m_s \int_{m_s^2}^{s_0} \frac{1}{s} (1-x) \left[1 + \frac{(1-x)(2-5x)}{3x}\right] e^{-\frac{s}{M_B^2}} ds - \frac{4}{3} a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds. \tag{49}
\end{aligned}$$

It is similar that the relative sign of λ_2 and λ_3 to λ_1 can be given by the following two sum rules:

$$(2\pi)^4 (\lambda_1 \lambda_2^* + \lambda_1^* \lambda_2) M^3 e^{-\frac{M^2}{M_B^2}} = -12m_s a \int_{m_s^2}^{s_0} (1+m_0^2/s)(1-x)^2 e^{-\frac{s}{M_B^2}} ds, \tag{50}$$

and

$$(2\pi)^4 (\lambda_1 \lambda_3^* + \lambda_1^* \lambda_3) M^3 e^{-\frac{M^2}{M_B^2}} = - \int_{m_s^2}^{s_0} \{a s (1-x)^2 (2+x)$$

$$+\frac{m_0^2 a}{2}\left[1-\frac{3}{2}(1-x)(1+x)+(1-x)(13-25x+2x^2)\right]e^{-\frac{s}{M_B^2}}ds. \quad (51)$$

Fig. 2 gives the dependence of the above sum rules on the Borel parameter M_B^2 .

The final numerical values of the coupling constants of Σ are:

$$\begin{aligned} f_\Sigma &= (9.4 \pm 0.4) \times 10^{-3} \text{ GeV}^2, & \lambda_1 &= -(2.5 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\ \lambda_2 &= (4.4 \pm 0.1) \times 10^{-2} \text{ GeV}^2, & \lambda_3 &= (2.0 \pm 0.1) \times 10^{-2} \text{ GeV}^2. \end{aligned} \quad (52)$$

In Ref. [9], Chernyak *et al.* have calculated nonperturbative parameters relevant to leading twist contributions. Their numerical results are $|f_\Sigma| \simeq 0.51 \times 10^{-2} \text{ GeV}^2$ and $|f_\Sigma^T| \simeq 0.49 \times 10^{-2} \text{ GeV}^2$. By contrast, we use Eqs. (32) and (33) to give our estimations: $|f_\Sigma| = 0.94 \times 10^{-2} \text{ GeV}^2$ and $|f_\Sigma^T| = 0.60 \times 10^{-2} \text{ GeV}^2$. In comparison with their results, our predictions are larger in absolute values.

4.2 QCD sum rules for Λ baryon

The sum rules of the Λ baryon parameters begin with the following correlation functions:

$$\Pi_i(q^2) = \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_i(x) \bar{j}_i(0) \} | 0 \rangle, \quad (53)$$

with the definitions of the currents:

$$j_1(x) = \epsilon^{ijk} [u^i(x) C \gamma_5 \not{z} d^j(x)] \not{z} s^k(x), \quad (54)$$

$$j_2(x) = \epsilon^{ijk} [u^i(x) C \gamma_5 \gamma_\mu d^j(x)] \gamma^\mu s^k(x), \quad (55)$$

$$j_3(x) = \epsilon^{ijk} [u^i(x) C \gamma_5 d^j(x)] s^k(x), \quad (56)$$

$$j_4(x) = \epsilon^{ijk} [u^i(x) C d^j(x)] \gamma_5 s^k(x). \quad (57)$$

The similar processes as in the above subsection lead to the following results:

$$(4\pi)^4 f_\Lambda^2 e^{-\frac{M^2}{M_B^2}} = \frac{2}{5} \int_{m_s^2}^{s_0} s(1-x)^5 e^{-\frac{s}{M_B^2}} ds - \frac{b}{3} \int_{m_s^2}^{s_0} \frac{1}{s} x(1-x)(1-2x) e^{-\frac{s}{M_B^2}} ds, \quad (58)$$

and

$$4(2\pi)^4 \lambda_1^2 M^2 e^{-\frac{M^2}{M_B^2}} = \frac{1}{2} \int_{m_s^2}^{s_0} s^2 [(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] e^{-\frac{s}{M_B^2}} ds$$

$$\begin{aligned}
& + \frac{b}{12} \int_{m_s^2}^{s_0} (1-x)^2 e^{-\frac{s}{M_B^2}} ds - \frac{4}{3} a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4}\right. \\
& \left. + \frac{m_0^4 m_s^4}{16M_B^8}\right) e^{-\frac{m_s^2}{M_B^2}} - m_s a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds,
\end{aligned} \tag{59}$$

and

$$\begin{aligned}
4(4\pi)^4 \lambda_2^2 M^2 e^{-\frac{M^2}{M_B^2}} &= \int_{m_s^2}^{s_0} s^2 [(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] e^{-\frac{s}{M_B^2}} ds \\
&+ \frac{b}{3} \int_{m_s^2}^{s_0} (1-x)(1+5x) e^{-\frac{s}{M_B^2}} ds + \frac{32}{3} a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4}\right. \\
&\left. + \frac{m_0^4 m_s^4}{16M_B^8}\right) e^{-\frac{m_s^2}{M_B^2}} - 4m_s a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds,
\end{aligned} \tag{60}$$

and

$$\begin{aligned}
4(4\pi)^4 \lambda_3^2 M^2 e^{-\frac{M^2}{M_B^2}} &= \int_{m_s^2}^{s_0} s^2 [(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] e^{-\frac{s}{M_B^2}} ds \\
&+ \frac{b}{3} \int_{m_s^2}^{s_0} (1-x)(1+5x) e^{-\frac{s}{M_B^2}} ds + \frac{32}{3} a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4}\right. \\
&\left. + \frac{m_0^4 m_s^4}{16M_B^8}\right) e^{-\frac{m_s^2}{M_B^2}} - 4m_s a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds,
\end{aligned} \tag{61}$$

and the sum rule of $f_\Lambda \lambda_1^*$ is

$$\begin{aligned}
(4\pi)^4 f_\Lambda \lambda_1^* M e^{-\frac{M^2}{M_B^2}} &= \frac{2}{3} m_s \int_{m_s^2}^{s_0} s [(1-x)(3+13x-5x^2+x^3) + 12x \ln x] e^{-\frac{s}{M_B^2}} ds \\
&+ \frac{b}{3} m_s \int_{m_s^2}^{s_0} \frac{1}{s} (1-x) \left[1 + \frac{(1-x)(2-5x)}{3x}\right] e^{-\frac{s}{M_B^2}} ds + \frac{8}{3} a_s \int_{m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds.
\end{aligned} \tag{62}$$

Note that the sum rules of λ_2 and λ_3 are the same. In Fig. 3, the sum rules of the parameters on the Borel parameter M_B^2 are shown. The final numerical results for the parameters of Λ are:

$$\begin{aligned}
f_\Lambda &= (6.0 \pm 0.3) \times 10^{-3} \text{ GeV}^2, & \lambda_1 &= (1.0 \pm 0.3) \times 10^{-2} \text{ GeV}^2, \\
|\lambda_2| &= (0.83 \pm 0.05) \times 10^{-2} \text{ GeV}^2, & |\lambda_3| &= (0.83 \pm 0.05) \times 10^{-2} \text{ GeV}^2.
\end{aligned} \tag{63}$$

In the above results, f_Λ and λ_1 have the same sign, which is different from that shown in Ref. [11]. The relative sign of λ_1 and λ_2 cannot be determined by the method presented above. Here we only list the absolute values of the two parameters.

The numerical estimations by Chernyak *et al.* are: $|f_\Lambda| \simeq 0.63 \times 10^{-2} \text{ GeV}^2$ and $|f_\Lambda^T| \simeq 0.063 \times 10^{-2} \text{ GeV}^2$. Our result on f_Λ is $f_\Lambda = 0.60 \times 10^{-2} \text{ GeV}^2$, but in our calculations f_Λ^T is zero, which is different from that of Chernyak. The deviation is due to the tensor structure of the baryon, which disappears at the leading order of the conformal expansion because of the isospin symmetry. In our approach, the determination of the tensor coupling constant relies on the next conformal expansion of the DAs, which corresponds to the sum rules of higher order moments in Ref. [9].

5 Application:electromagnetic form factors of the baryons with light-cone QCD sum rules

5.1 LCSR for the electromagnetic form factors

The EM form factors of hadrons are fundamental objects for understanding their internal structures. There were a lot of investigations on various hadrons both experimentally and theoretically, including meson [24, 25, 26, 27, 28, 29] and baryon [30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. While as there were few experimental data and theoretical investigations, the EM form factors of the baryons, such as Σ and Λ and so on, have not received much attention in the past years. Chiral perturbation theory and the chiral quark/soliton model have been used to study the baryon EM form factors at low momentum transfer [40, 41]. T. Van Cauteren *et al.* have investigated the electric and magnetic form factors of these baryons in the relativistic constituent quark model [42]. In the previous work [43] we gave an investigation on the EM form factors of the Λ baryon. In this section, the EM form factors of the Σ baryon are investigated at moderately large momentum transfer within the framework of light-cone QCD sum rule method, and the magnetic moments of the same baryons are estimated by comparing our results with the existing dipole formula.

The matrix element of the electromagnetic current between the baryon states can be expressed as the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$, respectively:

$$\langle \Sigma(P, s) | j_\mu^{em}(0) | \Sigma(P', s') \rangle = \bar{\Sigma}(P, s) [\gamma_\mu F_1(Q^2) - i \frac{\sigma_{\mu\nu} q^\nu}{2M} F_2(Q^2)] \Sigma(P', s'), \quad (64)$$

where $j_\mu^{em} = e_u \bar{u} \gamma_\mu u + e_s \bar{s} \gamma_\mu s$ is the electromagnetic current relevant to the hadron, and P, s and P', s' are the four-momenta and the spins of the initial and the final Σ baryon states, respectively. From the experimental viewpoint, the EM form factors can be expressed by the electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ Sachs form factors:

$$\begin{aligned} G_M(Q^2) &= F_1(Q^2) + F_2(Q^2), \\ G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \end{aligned} \quad (65)$$

and at the point $Q^2 = 0$ the magnetic $G_M(Q^2)$ form factor gives the magnetic moment of the baryon:

$$G_M(0) = \mu_\Sigma. \quad (66)$$

In order to evaluate the magnetic moment of the baryon from its EM form factors, the magnetic form factor $G_M(Q^2)$ is assumed to be described by the dipole formula:

$$\frac{1}{\mu_\Sigma} G_M(Q^2) = \frac{1}{(1 + Q^2/m_0^2)^2} = G_D(Q^2). \quad (67)$$

As there is no information about the parameter m_0^2 from experimental data, the two parameters m_0^2 and μ_Σ are estimated simultaneously by fitting the magnetic form factor with the dipole formula (67).

The calculation mainly focuses on Σ^+ baryon and it is similar for the calculation of Σ^- . The process of the derivation begins with the correlation function:

$$T_\mu(P, q) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_{\Sigma^+}(0) j_\mu^{em}(x) \} | \Sigma^+(P, s) \rangle, \quad (68)$$

where the interpolating current j_{Σ^+} is chosen as Eq. (40). The hadronic representation of the correlation function is acquired by inserting a complete set of states with the same quantum numbers as those of Σ^+ :

$$\begin{aligned} z^\mu T_\mu(P, q) &= \frac{1}{M^2 - P'^2} f_{\Sigma^+}(P' \cdot z) [2(P' \cdot z F_1(Q^2) - \frac{q \cdot z}{2} F_2(Q^2)) \not{z} \\ &\quad + (P' \cdot z F_2(Q^2) + \frac{q \cdot z}{2} F_2(Q^2)) \frac{\not{z} \not{q}}{M}] \Sigma^+(P, s) + \dots, \end{aligned} \quad (69)$$

where $P' = P - q$, and the dots stand for the higher resonances and continuum contributions. The correlation function is contracted with z^μ to get rid of contributions proportional to z^μ which is subdominant on the light cone. On the theoretical side, the correlation function (68) can be calculated to leading order of α_s as

$$\begin{aligned}
z_\mu T^\mu = & (P \cdot z)^2 (\not{z} \Sigma^+)_\gamma \left\{ 4e_u \int_0^1 d\alpha_2 \frac{1}{s - p'^2} \left\{ B_0(\alpha_2) + \frac{M^2}{(s - p'^2)} B_1(\alpha_2) \right. \right. \\
& - 2 \frac{M^4}{(s - P'^2)^2} B_2(\alpha_2) \left. \right\} + 2e_s \int_0^1 d\alpha_3 \frac{1}{s_2 - P'^2} \left\{ C_0(\alpha_3) \right. \\
& + \frac{M^2}{(s_2 - P'^2)} C_1(\alpha_3) - 2 \frac{M^4}{(s_2 - P'^2)^2} C_2(\alpha_3) \left. \right\} \left. \right\} \\
& + (P \cdot z)^2 M (\not{z} \not{q} \Sigma^+)_\gamma \left\{ 4e_u \int_0^1 d\alpha_2 \frac{1}{\alpha_2 (s - P'^2)^2} \left\{ -D_1(\alpha_2) \right. \right. \\
& + 2 \frac{M^2}{(s - P'^2)} B_2(\alpha_2) \left. \right\} + e_s \int_0^1 d\alpha_3 \frac{1}{\alpha_3 (s_2 - P'^2)^2} \left\{ -E_1(\alpha_3) \right. \\
& + 2 \frac{M^2}{(s_2 - P'^2)} C_2(\alpha_3) \left. \right\} \left. \right\}, \tag{70}
\end{aligned}$$

where $s = (1 - \alpha_2)M^2 + \frac{(1 - \alpha_2)}{\alpha_2} Q^2$ and $s_2 = (1 - \alpha_3)M^2 + \frac{(1 - \alpha_3)}{\alpha_3} Q^2 + \frac{m_s^2}{\alpha_3}$. Here the following notations are used for convenience:

$$\begin{aligned}
B_0(\alpha_2) &= \int_0^{1 - \alpha_2} d\alpha_1 V_1(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2), \\
B_1(\alpha_2) &= (2\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5)(\alpha_2), \\
B_2(\alpha_2) &= (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6)(\alpha_2), \\
C_0(\alpha_3) &= \int_0^{1 - \alpha_3} d\alpha_1 V_1(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3), \\
C_1(\alpha_3) &= (2\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5)(\alpha_3), \\
C_2(\alpha_3) &= (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6)(\alpha_3), \\
D_1(\alpha_2) &= (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3)(\alpha_2), \\
E_1(\alpha_3) &= (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3)(\alpha_3), \tag{71}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{V}_i(\alpha_2) &= \int_0^{\alpha_2} d\alpha'_2 \int_0^{1-\alpha'_2} d\alpha_1 V_i(\alpha_1, \alpha'_2, 1 - \alpha_1 - \alpha'_2), \\
\tilde{\tilde{V}}_i(\alpha_2) &= \int_0^{\alpha_2} d\alpha'_2 \int_0^{\alpha'_2} d\alpha''_2 \int_0^{1-\alpha''_2} d\alpha_1 V_i(\alpha_1, \alpha''_2, 1 - \alpha_1 - \alpha''_2), \\
\tilde{V}_i(\alpha_3) &= \int_0^{\alpha_3} d\alpha'_3 \int_0^{1-\alpha'_3} d\alpha_1 V_i(\alpha_1, 1 - \alpha_1 - \alpha'_3, \alpha'_3), \\
\tilde{\tilde{V}}_i(\alpha_3) &= \int_0^{\alpha_3} d\alpha'_3 \int_0^{\alpha'_3} d\alpha''_3 \int_0^{1-\alpha''_3} d\alpha_1 V_i(\alpha_1, 1 - \alpha_1 - \alpha''_3, \alpha''_3).
\end{aligned} \tag{72}$$

Then equating both sides of the Borel transformed version of hadronic and theoretical representations with the assumption of quark-hadron duality, the final sum rules are given as follows:

$$\begin{aligned}
2f_{\Sigma^+}F_1(Q^2)e^{-\frac{M^2}{M_B^2}} &= 4e_u \int_{\alpha_{20}}^1 d\alpha_2 e^{-\frac{s}{M_B^2}} \left\{ B_0(\alpha_2) + \frac{M^2}{M_B^2} B_1(\alpha_2) - \frac{M^4}{M_B^4} B_2(\alpha_2) \right\} \\
&+ 4e_u e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{20}^2 M^2}{\alpha_{20}^2 M^2 + Q^2} \left\{ B_1(\alpha_{20}) - \frac{M^2}{M_B^2} B_2(\alpha_{20}) \right\} \\
&+ 4e_u e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{20}^2 M^4}{\alpha_{20}^2 M^2 + Q^2} \frac{d}{d\alpha_{20}} B_2(\alpha_{20}) \frac{\alpha_{20}^2}{\alpha_{20}^2 M^2 + Q^2} \\
&+ 2e_s \int_{\alpha_{30}}^1 d\alpha_3 e^{-\frac{s}{M_B^2}} \left\{ C_0(\alpha_3) + \frac{M^2}{M_B^2} C_1(\alpha_3) - \frac{M^4}{M_B^4} C_2(\alpha_3) \right\} \\
&+ 2e_s e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{30}^2 M^2}{\alpha_{30}^2 M^2 + Q^2 + m_s^2} \left\{ C_1(\alpha_{30}) - \frac{M^2}{M_B^2} C_2(\alpha_{30}) \right\} \\
&+ 2e_s e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{30}^2 M^4}{\alpha_{30}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{30}} C_2(\alpha_{30}) \frac{\alpha_{30}^2}{\alpha_{30}^2 M^2 + Q^2 + m_s^2}
\end{aligned} \tag{73}$$

for $F_1(Q^2)$ and

$$\begin{aligned}
f_{\Sigma^+}F_2(Q^2)e^{-\frac{M^2}{M_B^2}} &= M^2 \left\{ 4e_u \int_{\alpha_{20}}^1 d\alpha_2 e^{-\frac{s}{M_B^2}} \frac{1}{\alpha_2 M_B^2} \left\{ -D_1(\alpha_2) + \frac{M^2}{M_B^2} B_2(\alpha_2) \right\} \right. \\
&- 4e_u e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{20}}{\alpha_{20}^2 M^2 + Q^2} \left\{ D_1(\alpha_{20}) - \frac{M^2}{M_B^2} B_2(\alpha_{20}) \right\} \\
&- 4e_u e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{20}^2 M^2}{\alpha_{20}^2 M^2 + Q^2} \frac{d}{d\alpha_{20}} B_2(\alpha_{20}) \frac{\alpha_{20}}{\alpha_{20}^2 M^2 + Q^2}
\end{aligned}$$

$$\begin{aligned}
& +2e_s \int_{\alpha_{30}}^1 d\alpha_{30} e^{-\frac{s_2}{M_B^2}} \frac{1}{\alpha_3 M_B^2} \left\{ -E_1(\alpha_3) + \frac{M^2}{M_B^2} C_2(\alpha_3) \right\} \\
& -2e_s e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{30}}{\alpha_{30}^2 M^2 + Q^2 + m_s^2} \left\{ E_1(\alpha_{30}) - \frac{M^2}{M_B^2} C_2(\alpha_{30}) \right\} \\
& -2e_s e^{-\frac{s_0}{M_B^2}} \frac{\alpha_{30}^2 M^2}{\alpha_{30}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{30}} C_2(\alpha_{30}) \frac{\alpha_{30}}{\alpha_{30}^2 M^2 + Q^2 + m_s^2} \left\{ \right.
\end{aligned} \tag{74}$$

for $F_2(Q^2)$.

5.2 Numerical analysis

In the numerical analysis, the continuum threshold is chosen as $s_0 = (2.65 - 2.85) \text{ GeV}^2$. The masses of the Σ baryons from Ref. [44] are $M_{\Sigma^+} = 1.189 \text{ GeV}$ and $M_{\Sigma^-} = 1.197 \text{ GeV}$. The parameters f_Σ and λ_1 are used as the central values in Eqs. (52). For the auxiliary Borel parameter M_B^2 , there should be a region where the sum rules are almost independent of it. To choose a platform for the Borel parameter, we should suppress both resonance contributions and the higher twist contributions simultaneously. Fig. 4 shows the dependence of the magnetic form factors on the Borel parameter at different points of Q^2 . Our results are acceptable in the range $2.0 \text{ GeV}^2 \leq M_B^2 \leq 4.0 \text{ GeV}^2$.

The estimation on the magnetic moment of the baryon comes from the fitting of the magnetic form factor by the dipole formula (67). In the following analysis the Borel parameter is chosen to be $M_B^2 = 3 \text{ GeV}^2$. Fig. 5 gives the dependence of the magnetic form factor $G_M(Q^2)$ on the momentum transfer at different points of the threshold s_0 . The figure shows that $G_M(Q^2)$ decreases with Q^2 , which is in accordance with the assumption in Eq. (67). To estimate the magnetic moment numerically, the magnetic form factor $G_M(Q^2)$ is fitted by the formula $\mu_\Sigma/(1 + Q^2/m_0^2)^2$, which is described by the dashed lines in Fig. 5. From the figures the magnetic moment of Σ^+ is given as $\mu_{\Sigma^+} = (3.13 \pm 0.10)\mu_N$, and the estimation of the other parameter is $m_0^2 = (0.86 \pm 0.04) \text{ GeV}^2$.

The similar process is carried out for the numerical analysis of Σ^- . The estimation of the Σ^- magnetic moment is shown in Fig. 6, which are the magnetic form factors of the

baryon at different threshold and the fittings by the dipole formula. The numerical values from the fittings are $\mu_{\Sigma^-} = -(1.59 \pm 0.02)\mu_N$ and $m_0^2 = (0.78 \pm 0.03) \text{ GeV}^2$.

Table 2 lists magnetic moments of the two baryons from various approaches: data from Particle Data Group (PDG) [44]; QCD sum rules [45] (SR(1) for $\chi = -3.3$ and SR(2) for $\chi = -4.5$); QCD string approach (QCDSA) [46]; chiral perturbation theory (χ PT) [47]; Skyrme model (SKRM) [48]; light cone sum rules [49] (LCSR(1) for $\chi = -3.3$ and LCSR(2) for $\chi = -4.5$). The table shows that our results are larger in absolute values than the others. This may partly lie in the fact that more detailed information on DAs calls for higher order conformal expansion. At the same time, the choice of the interpolating currents may affect the results to some extent [15, 16]. The estimation is expected to be better if more information about the DAs are known and higher order QCD coupling $O(\alpha_s)$ effect are included.

Finally, the Q^2 -dependence of the physical value $G_M/\mu_\Sigma G_D$ is given in Fig. 7. In the numerical analysis, the input parameters m_0^2 used in the dipole formula (67) are the central values obtained above, which are $m_0^2 = 0.86 \text{ GeV}^2$ for Σ^+ and $m_0^2 = 0.78 \text{ GeV}^2$ for Σ^- , while the magnetic moments used come from Ref. [44], which are $\mu_{\Sigma^+} = 2.458\mu_N$ and $\mu_{\Sigma^-} = -1.160\mu_N$.

6 Summary

In this paper we present the DAs of baryons with quantum number $I(J^P) = 1(\frac{1}{2}^+)$ (for Σ^\pm) and $I(J^P) = 0(\frac{1}{2}^+)$ (for Λ) up to twist 6. We find that fourteen independent DAs are needed to describe the structure of the baryons. The method employed is based on the conformal partial wave expansion, and the nonlocal nonperturbative parameters are determined in the QCD sum rule framework. Our calculation on the conformal expansion of the DAs is to leading order conformal spin accuracy. Compared with the previous work [11], the calculation on the Λ baryon gives DAs of all other Lorentz structures besides axial-like vector structures. Another new result is that the relative sign of the two parameters f_Λ and λ_1 are positive.

With the DAs obtained, the EM form factors of Σ are investigated in the range $1 \text{ GeV}^2 \leq Q^2 \leq 7 \text{ GeV}^2$. We assume that the magnetic form factor can be described by the dipole formula. Fitting the result by the dipole formula, the magnetic moments of the baryons are estimated, which are $\mu_{\Sigma^+} = (3.13 \pm 0.10)\mu_N$, and $\mu_{\Sigma^-} = -(1.59 \pm 0.02)\mu_N$. Compared with values given by Particle Data Group [44], our results are larger in absolute values. This shows that the calculation needs more detailed information on the DAs, which may come from higher order conformal spin contributions, and at the same time the choice of the interpolating currents may also affect our estimation to some extent.

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Appendices

In the appendices we give our results on the DAs of Σ and Λ explicitly. As the definition in Eq. (7), our results are listed in the following subsections.

A DAs of the Σ baryon

Twist-3 distribution amplitudes of Σ are:

$$\begin{aligned} V_1(x_i) &= 120x_1x_2x_3\phi_3^0, & A_1(x_i) &= 0, \\ T_1(x_i) &= 120x_1x_2x_3\phi_3^{\prime 0}. \end{aligned} \tag{75}$$

Twist-4 distribution amplitudes are:

$$\begin{aligned} S_1(x_i) &= 6(x_2 - x_1)x_3(\xi_4^0 + \xi_4^{\prime 0}), & P_1(x_i) &= 6(x_2 - x_1)x_3(\xi_4^0 - \xi_4^{\prime 0}), \\ V_2(x_i) &= 24x_1x_2\phi_4^0, & A_2(x_i) &= 0, \\ V_3(x_i) &= 12x_3(1 - x_3)\psi_4^0, & A_3(x_i) &= -12x_3(x_1 - x_2)\psi_4^0, \end{aligned}$$

$$\begin{aligned}
T_2(x_i) &= 24x_1x_2\phi_4^{'0}, & T_3(x_i) &= 6x_3(1-x_3)(\xi_4^0 + \xi_4^{'0}), \\
T_7(x_i) &= 6x_3(1-x_3)(\xi_4^{'0} - \xi_4^0). & & (76)
\end{aligned}$$

Twist-5 distribution amplitudes are:

$$\begin{aligned}
S_2(x_i) &= \frac{3}{2}(x_1 - x_2)(\xi_5^0 + \xi_5^{'0}), & P_2(x_i) &= \frac{3}{2}(x_1 - x_2)(\xi_5^0 - \xi_5^{'0}), \\
V_4(x_i) &= 3(1 - x_3)\psi_5^0, & A_4(x_i) &= 3(x_1 - x_2)\psi_5^0, \\
V_5(x_i) &= 6x_3\phi_5^0, & A_5(x_i) &= 0, \\
T_4(x_i) &= -\frac{3}{2}(x_1 + x_2)(\xi_5^{'0} + \xi_5^0), & T_5(x_i) &= 6x_3\phi_5^{'0}, \\
T_8(x_i) &= \frac{3}{2}(x_1 + x_2)(\xi_5^{'0} - \xi_5^0). & & (77)
\end{aligned}$$

Finally twist-6 distribution amplitudes are:

$$\begin{aligned}
V_6(x_i) &= 2\phi_6^0, & A_6(x_i) &= 0, \\
T_6(x_i) &= 2\phi_6^{'0}. & & (78)
\end{aligned}$$

B DAs of the Λ baryon

Twist-3 distribution amplitudes of Λ are:

$$\begin{aligned}
V_1(x_i) &= 0, & A_1(x_i) &= -120x_1x_2x_3\phi_3^0, \\
T_1(x_i) &= 0. & & (79)
\end{aligned}$$

Twist-4 distribution amplitudes are:

$$\begin{aligned}
S_1(x_i) &= 6x_3(1-x_3)(\xi_4^0 + \xi_4^{'0}), & P_1(x_i) &= 6(1-x_3)(\xi_4^0 - \xi_4^{'0}), \\
V_2(x_i) &= 0, & A_2(x_i) &= -24x_1x_2\phi_4^0, \\
V_3(x_i) &= 12(x_1 - x_2)x_3\psi_4^0, & A_3(x_i) &= -12x_3(1-x_3)\psi_4^0, \\
T_2(x_i) &= 0, & T_3(x_i) &= 6(x_2 - x_1)x_3(-\xi_4^0 + \xi_4^{'0}), \\
T_7(x_i) &= -6(x_1 - x_2)x_3(\xi_4^0 + \xi_4^{'0}). & & (80)
\end{aligned}$$

Twist-5 distribution amplitudes are:

$$\begin{aligned}
S_2(x_i) &= \frac{3}{2}(x_1 + x_2)(\xi_5^0 + \xi_5'^0), & P_2(x_i) &= \frac{3}{2}(x_1 + x_2)(\xi_5^0 - \xi_5'^0), \\
V_4(x_i) &= 3(x_2 - x_1)\psi_5^0, & A_4(x_i) &= -3(1 - x_3)\psi_5^0, \\
V_5(x_i) &= 0, & A_5(x_i) &= -6x_3\phi_5^0, \\
T_4(x_i) &= -\frac{3}{2}(x_1 - x_2)(\xi_5^0 + \xi_5'^0), & T_5(x_i) &= 0, \\
T_8(x_i) &= -\frac{3}{2}(x_1 - x_2)(\xi_5^0 - \xi_5'^0). & &
\end{aligned} \tag{81}$$

Finally twist-6 distribution amplitudes are:

$$\begin{aligned}
V_6(x_i) &= 0, & A_6(x_i) &= -2\phi_6^0, \\
T_6(x_i) &= 0. & &
\end{aligned} \tag{82}$$

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Fig. 1. Dependence of the four parameters f_Σ , λ_1 , λ_2 and λ_3 of Σ on the Borel parameter M_B^2 . The lines correspond to the threshold $s_0 = 2.65 - 2.85 \text{ GeV}^2$ from the bottom up.

Fig. 2. Sum rules of the relative signs of the parameters on the Borel parameter. The threshold is used as $s_0 = 1.66^2 \text{ GeV}^2$.

Fig. 3. Dependence of the four parameters f_Λ , λ_1 , λ_2 and $f_\Lambda \lambda_1^*$ of Λ on the Borel parameter M_B^2 . The lines correspond to the threshold $s_0 = 2.45 - 2.65 \text{ GeV}^2$ from the bottom up.

Fig. 4. Dependence of the form factor $G_M(Q^2)$ of Σ on the Borel parameter at different momentum transfer. The lines correspond to the points $Q^2 = 1, 2, 3, 5, 6 \text{ GeV}^2$ from the up down (left Σ^+) and from the bottom up (right Σ^-), respectively.

Fig. 5. Fittings of the form factor $G_M(Q^2)$ by the dipole formula $\mu_{\Sigma^+}/(1 + Q^2/m_0^2)^2$. The dashed lines are the fittings, and figures (a), (b) correspond to the threshold $s_0 = 2.65, 2.85 \text{ GeV}^2$, respectively.

Fig. 6. Fittings of the form factor $G_M(Q^2)$ by the dipole formula $\mu_{\Sigma^-}/(1 + Q^2/m_0^2)^2$. The dashed lines are the fittings, and figures (a), (b) correspond to the threshold $s_0 = 2.65, 2.85 \text{ GeV}^2$, respectively.

Fig. 7. The Q^2 -dependence of the form factor $G_M/(\mu_\Sigma G_D)$. The lines correspond to the threshold $s_0 = 2.65, 2.75, 2.85 \text{ GeV}^2$ from the bottom up. The left corresponds to Σ^+ and the right corresponds to Σ^- .

Table 1: Independent baryon distribution amplitudes that enter the expansion in Eqs. (17) to (22).

	Lorentz-structure	Light-cone projection	nomenclature
twist-3	$(C \not{z}) \otimes \not{z}$	$u_{\uparrow}^+ u_{\downarrow}^+ s_{\uparrow}^+$	$\Phi_3(x_i) = [V_1 - A_1](x_i)$
	$(Ci\sigma_{\perp z}) \otimes \gamma^{\perp} \not{z}$	$u_{\uparrow}^+ u_{\uparrow}^+ s_{\downarrow}^+$	$T_1(x_i)$
twist-4	$(C \not{z}) \otimes \not{p}$	$u_{\uparrow}^+ u_{\downarrow}^+ s_{\uparrow}^-$	$\Phi_4(x_i) = [V_2 - A_2](x_i)$
	$(C \not{z} \gamma_{\perp} \not{p}) \otimes \gamma^{\perp} \not{z}$	$u_{\uparrow}^+ u_{\downarrow}^- s_{\downarrow}^+$	$\Psi_4(x_i) = [V_3 - A_3](x_i)$
	$(C \not{p} \not{z}) \otimes \not{z}$	$u_{\uparrow}^- u_{\uparrow}^+ s_{\uparrow}^+$	$\Xi_4(x_i) = [T_3 - T_7 + S_1 + P_1](x_i)$
	$(C \not{p} \not{z}) \otimes \not{z}$	$u_{\downarrow}^- u_{\uparrow}^+ s_{\uparrow}^+$	$\Xi'_4(x_i) = [T_3 + T_7 + S_1 - P_1](x_i)$
	$(Ci\sigma_{\perp z}) \otimes \gamma^{\perp} \not{p}$	$u_{\downarrow}^+ u_{\downarrow}^+ s_{\downarrow}^-$	$T_2(x_i)$
twist-5	$(C \not{p}) \otimes \not{z}$	$u_{\uparrow}^- u_{\downarrow}^- s_{\uparrow}^+$	$\Phi_5(x_i) = [V_5 - A_5](x_i)$
	$(C \not{p} \gamma_{\perp} \not{z}) \otimes \gamma^{\perp} \not{p}$	$u_{\uparrow}^- u_{\downarrow}^+ s_{\downarrow}^-$	$\Psi_5(x_i) = [V_4 - A_4](x_i)$
	$(C \not{z} \not{p}) \otimes \not{p}$	$u_{\uparrow}^+ u_{\uparrow}^- s_{\uparrow}^-$	$\Xi'_5(x_i) = [-T_4 - T_8 + S_2 + P_2](x_i)$
	$(C \not{z} \not{p}) \otimes \not{p}$	$u_{\downarrow}^+ u_{\downarrow}^- s_{\uparrow}^-$	$\Xi_5(x_i) = [S_2 - P_2 - T_4 + T_8](x_i)$
	$(Ci\sigma_{\perp p}) \otimes \gamma^{\perp} \not{z}$	$u_{\downarrow}^- u_{\downarrow}^- s_{\downarrow}^+$	$T_5(x_i)$
twist-6	$(C \not{p}) \otimes \not{p}$	$u_{\uparrow}^- u_{\downarrow}^- s_{\uparrow}^-$	$\Phi_6(x_i) = [V_6 - A_6](x_i)$
	$(Ci\sigma_{\perp p}) \otimes \gamma^{\perp} \not{p}$	$u_{\uparrow}^- u_{\uparrow}^- s_{\downarrow}^-$	$T_6(x_i)$

Table 2: Magnetic moments of the Σ baryons from various models

$\mu(\mu_N)$	PDG	SR(1)	SR(2)	QCDSA	χ PT	SKRM	LCSR(1)	LCSR(2)	Ours
μ_{Σ^+}	2.46	2.52	3.30	2.48	2.458	2.41	2.2	2.9	3.13
μ_{Σ^-}	-1.16	-1.13	-1.38	-0.90	-1.16	-1.10	-0.8	-1.1	-1.59

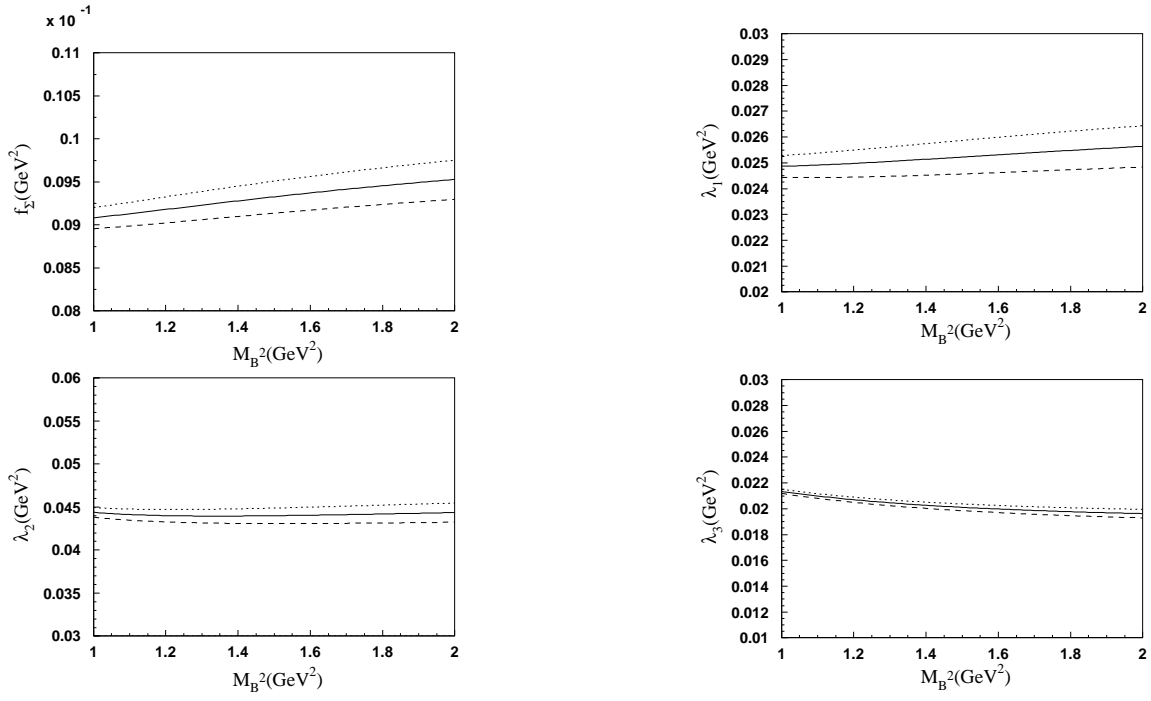


Figure 1:

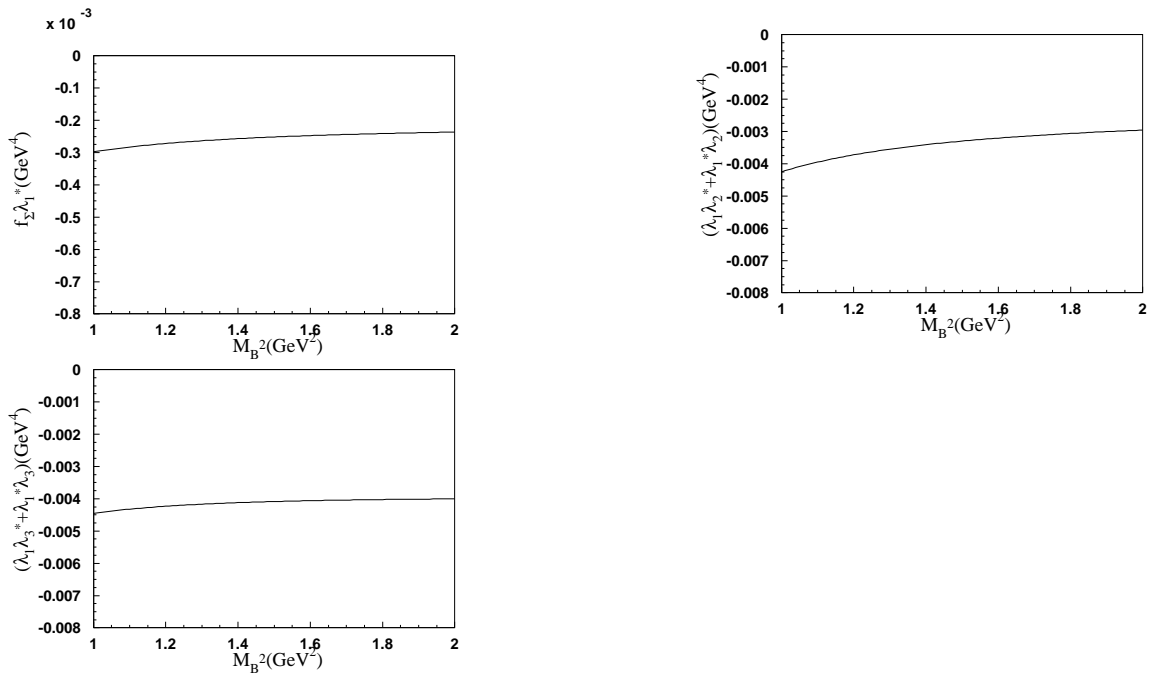


Figure 2:

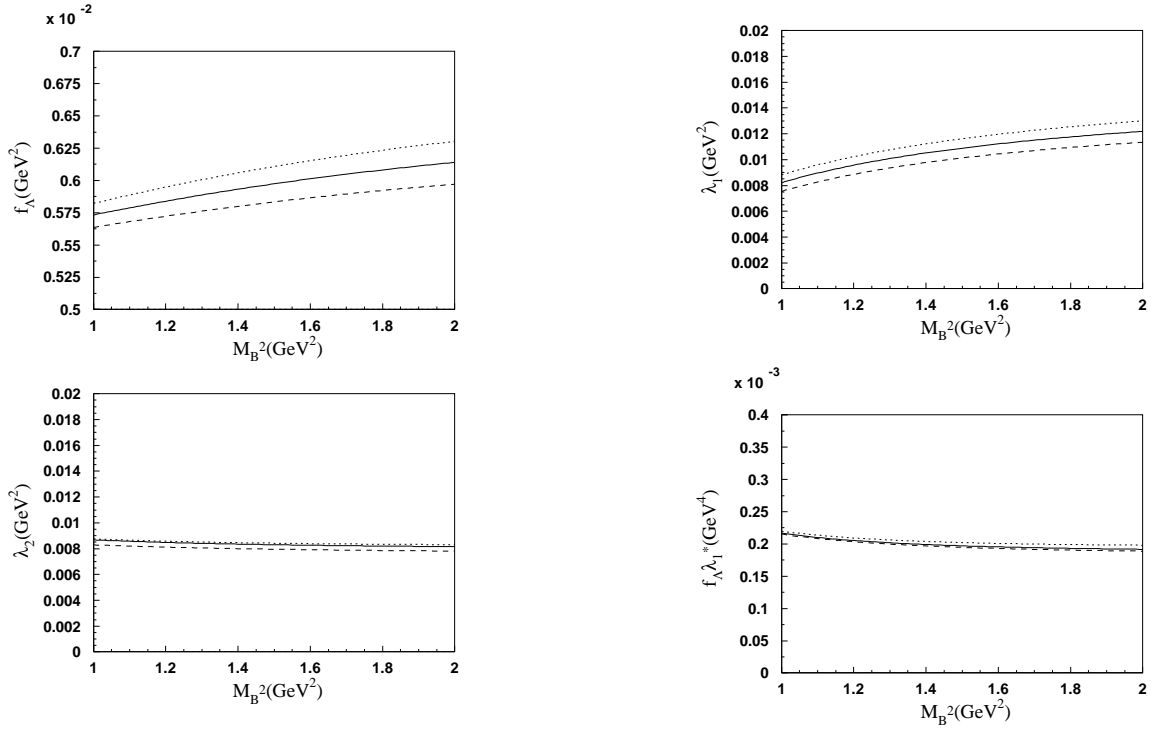


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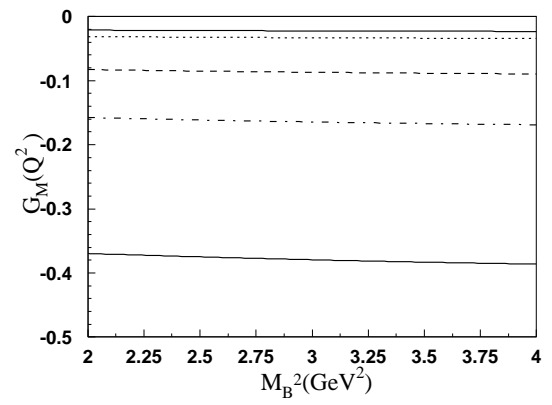
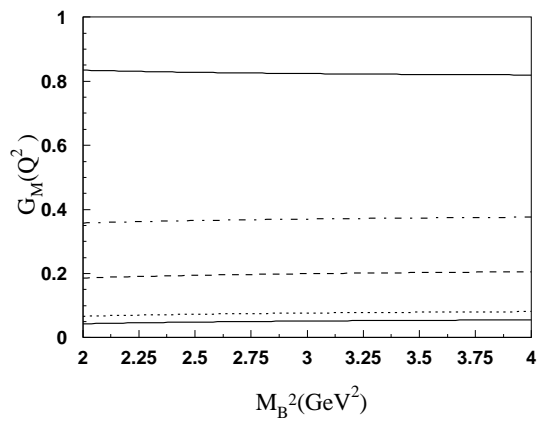


Figure 4:

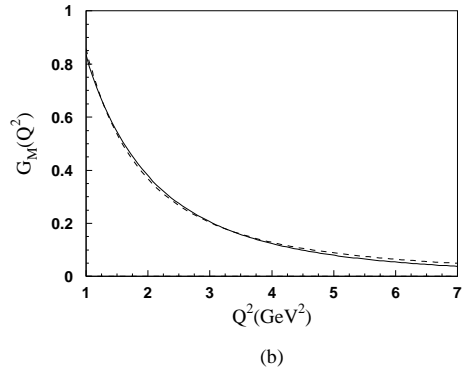
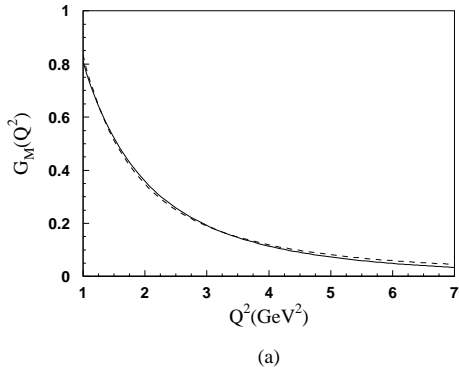


Figure 5:

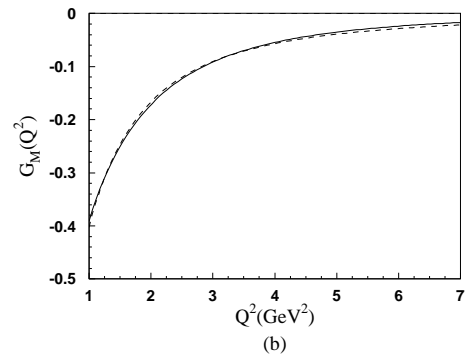
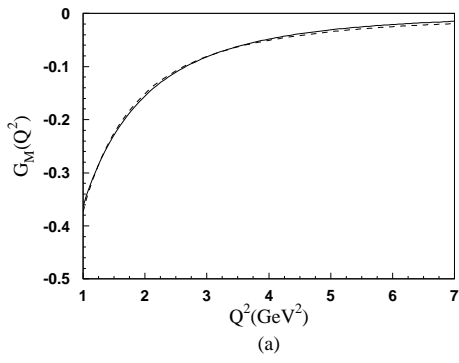


Figure 6:

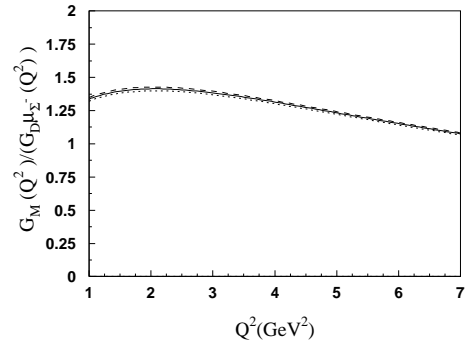
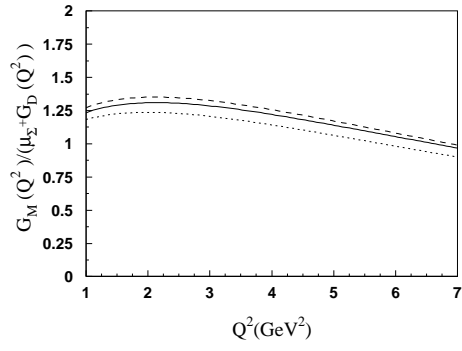


Figure 7: